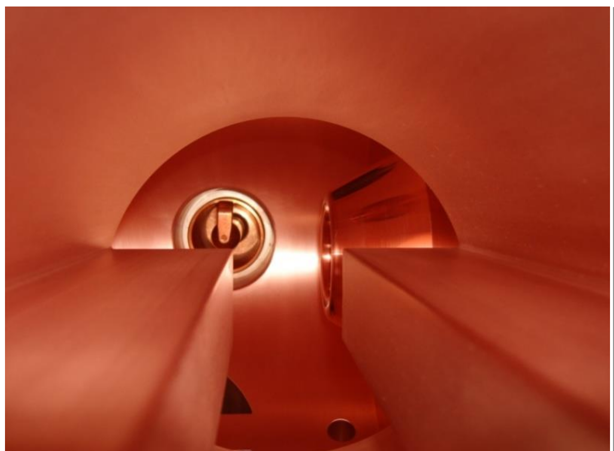
WP2 Mini-Workshop #1

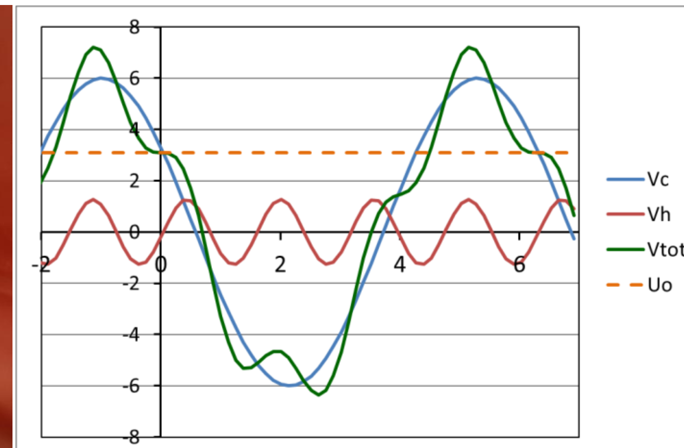
Beam Dynamic Simulations with harmonic RF

By Zoom, 23 June 2021

DC Robinson Instability with an Active NC Harmonic Cavity - Anticipation of Future Simulations-

Jörn Jacob



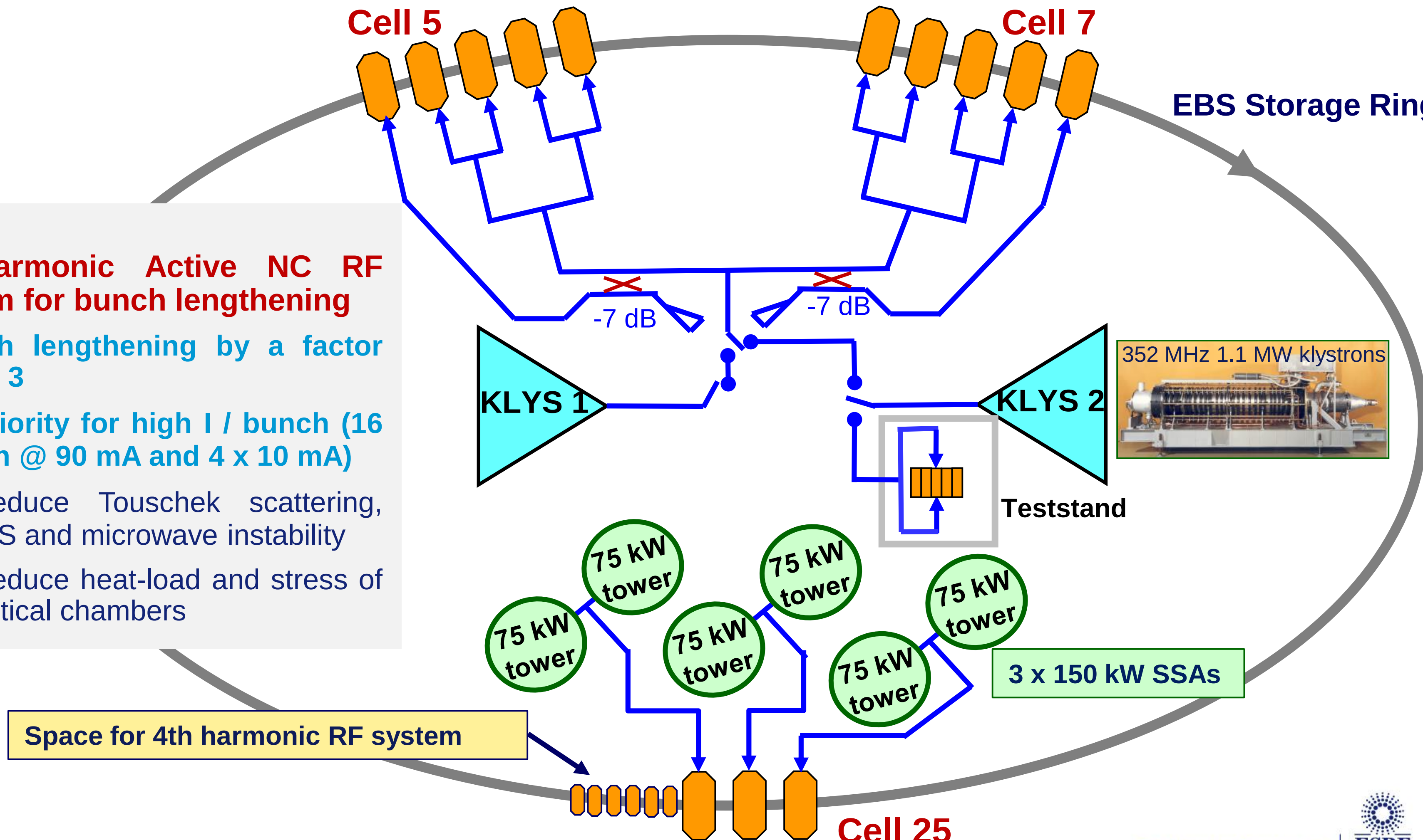


ESRF | The European Synchrotron

EBS 352 MHz RF SYSTEM LAYOUT

4th harmonic Active NC RF system for bunch lengthening

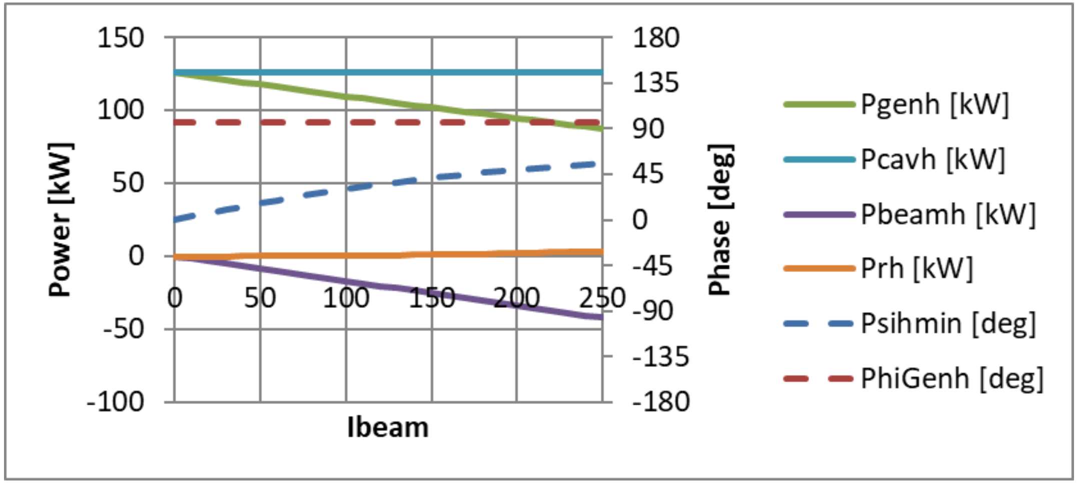
- **Bunch lengthening by a factor 2.5 to 3**
- **1st Priority for high I / bunch (16 bunch @ 90 mA and 4 x 10 mA)**
  - Reduce Touschek scattering, IBS and microwave instability
  - Reduce heat-load and stress of critical chambers



4^{TH} HARMONIC 1.41 GHz RF POWER REQUIREMENTS

- **Optimum bunch lengthening**
  - V_{acc} = 6.5 MV at 352.37 MHz (worst case, as V_{nom} = 6.0 MV)
  - All ID gaps open (2.56 MeV/turn, worst case)
  - ⇒ $V_{h,opt}$ = 1.49 MV at 1409 MHz

- **Harmonic cavity parameters**
  - 6 to 8 cells in TM020 mode
  - Total R_s = 6.6 to 8.8 MΩ
  - Coupling β_h = 1 (match)
  - Optimum Tuning for zero load angle (as for accelerating cavity)



Example: 8 cells in operation

BEAM LOADING DIAGRAM WITH HARMONIC CAVITY FOR BUNCH LENGTHENING

$$V_{acc}(\phi) = V_c \sin(\phi_s + \phi) + V_h \sin(n\phi_h + n\phi)$$

Optimum Working point (1st & 2nd derivatives = 0):

$$\phi_s = \pi - \arcsin[\, n^2/(n^2-1)\;\; U_0/V_c \,]$$

$$V_{h,opt} = \mathrm{sqrt}[\, V_c^2/n^2 - U_0^2/(n^2-1) \,]$$

$$\phi_{\,h,opt} = (1/n)\arcsin[- U_0 / (V_{h,opt}\,(n^2-1)) \,]$$

Optimum tuning (min power) ⇔ load angle = 0:

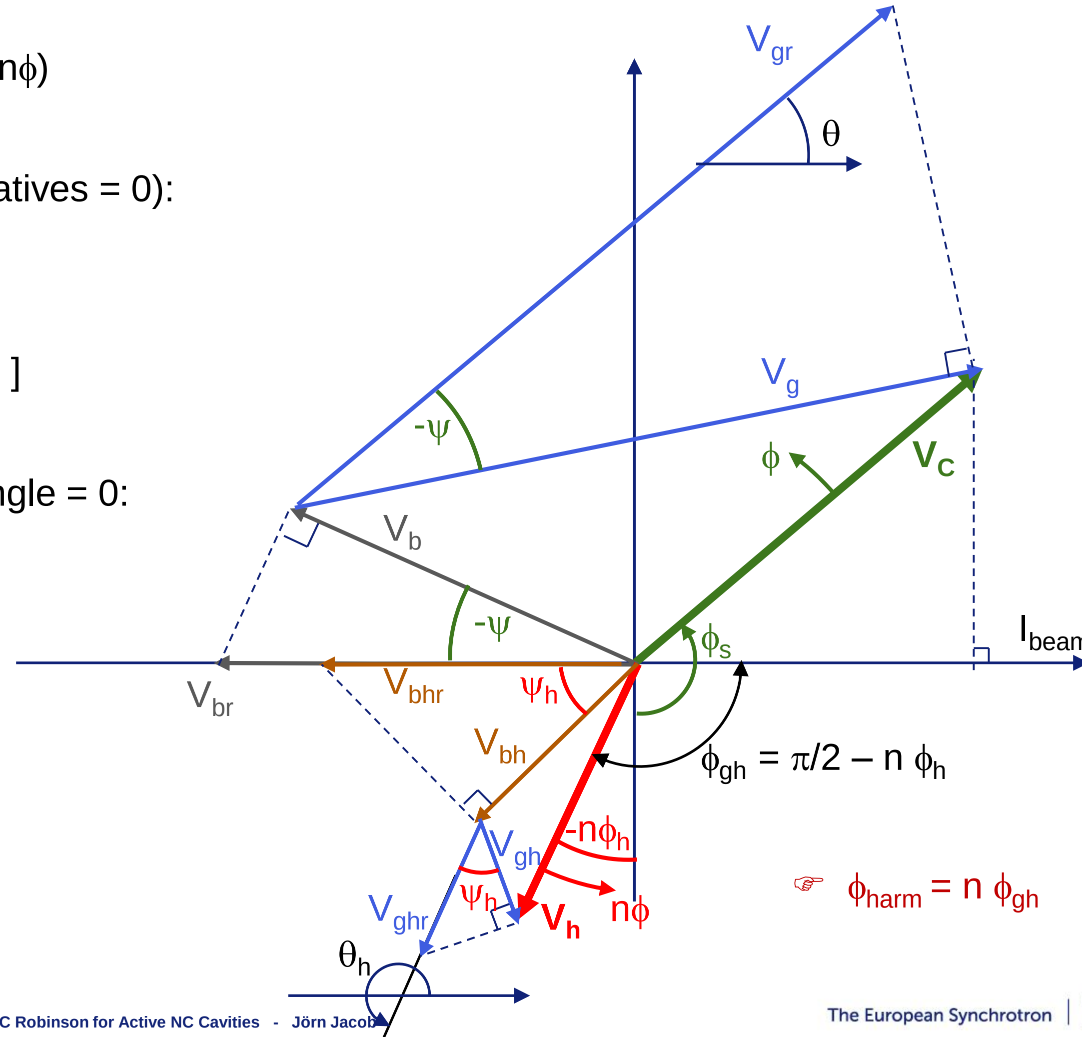
ψ such that V_{gr} // V_c

ψ_h such that V_{ghr} // V_h

Beware, in the vector diagram:

Main RF turns at $\phi = \omega t$

Harmonic RF at $n\phi = n\omega t$



Assumptions:

RF loops (Amp, Phi, tuning	slower than	Synchrotron motion	slower than	Cavity Bandwidths (main & HC)
B ≈ 1 Hz	<<	fs ≈ 1 kHz …	<<	Above ≈ 40 kHz

⇓

1. Tuning angles, generator amplitudes and phases are constant at the scale of the synchrotron motion

2. The beam induced voltages in the cavities follow the beam phase

$f_s = f_{rf} \times \mathrm{sqrt}\,[\,\alpha\, \mathbf{K'} / (2\pi\, h\, E_0/e)\,]$, (K'<0 ⇔ DC Robinson instability)

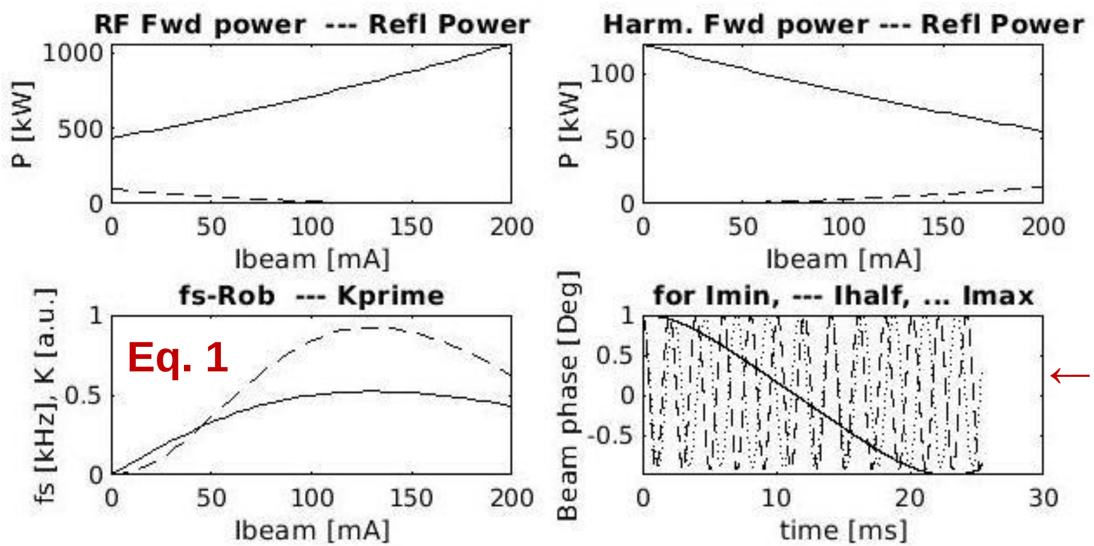
$$\mathbf{K'} = \underbrace{-V_c \cos\phi_s}_{\substack{>0 \\ \text{Main RF,} \\ \text{giving } f_{s0}}} \;\underbrace{-\, nV_h \cos(n\phi_h)}_{\substack{<0 \\ \text{Harm. RF, for} \\ \text{cancelling } f_s}} \;\underbrace{+\, V_b \sin\psi}_{\substack{<0 \\ \text{Main RF} \\ \text{beam loading} \\ \text{(Robinson term)}}} \;\underbrace{+\, nV_{bh} \sin\psi_h}_{\substack{>0 \\ \text{Harm. RF,} \\ \text{beam loading} \\ \text{(Stabilizing effect)}}} \qquad \textbf{(Eq. 1)}$$

Coming examples already shown
at
ESLS RF meeting at SOLEIL in November 2018

Computed for 3rd harmonic RF system
(and not for actual 4th harmonic project)

Numerical integration of synchrotron equation:

- Uniform filling (no transients)
- Starting with beam phase offset by +1 or -1 deg
- Tracking V_b, V_{bh} and ϕ_{beam} turn by turn
- Checking convergence (neglecting synchrotron oscillation damping)
- No linearization !



← Numerical integration

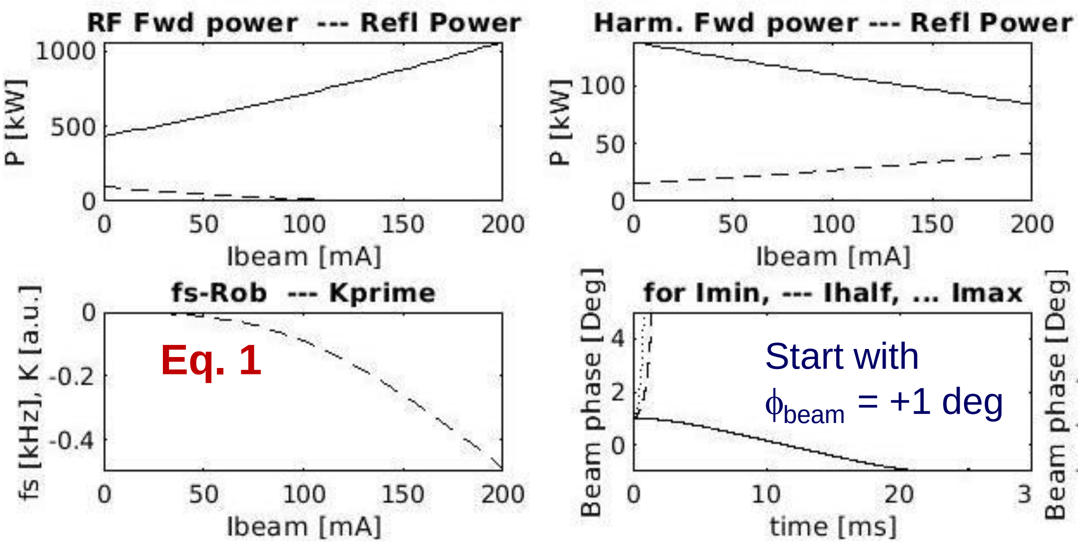
$V_h = V_{h,opt} = 1.89$ MV

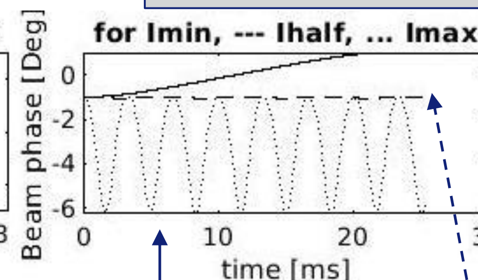
$n\phi_h = n\phi_{h,opt} = -12.2$ deg

5 cavities, $\beta_h = 1$

$\rightarrow$ stable

ROBINSON DC (2ND TYPE) – INTEGRATION OF SYNCHROTRON EQUATION





$V_h = V_{h,opt} = 1.89$ MV

$n\phi_h = n\phi_{h,opt} = -12.2$ deg

5 cavities, $\beta_h = 2$

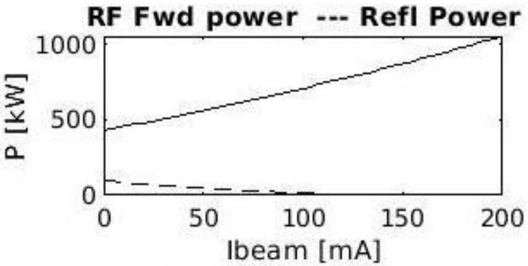
$\rightarrow$ Unstable for $I_{beam} > 0$

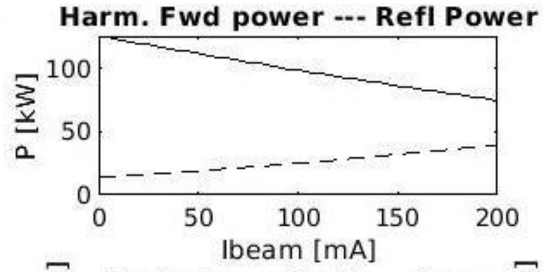
Start with
ϕ_{beam} = -1 deg

Only for negative beam phases: stabilization through non-linearity of voltage waveform

Equilibrium for ≈ 100 mA

ROBINSON DC (2^{ND} TYPE) – INTEGRATION OF SYNCHROTRON EQUATION



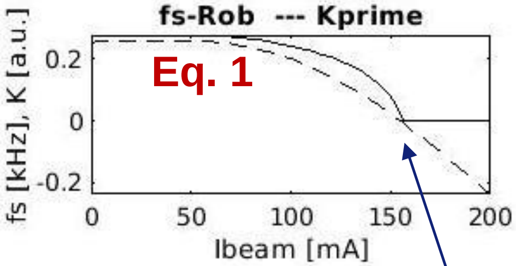


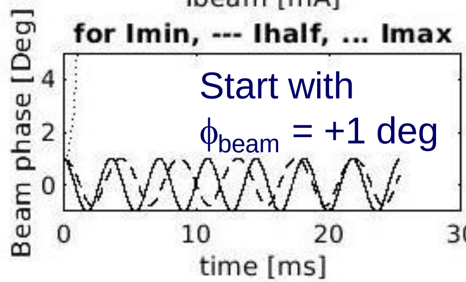
V_h = 1.80 MV ($\neq V_{h,opt}$)

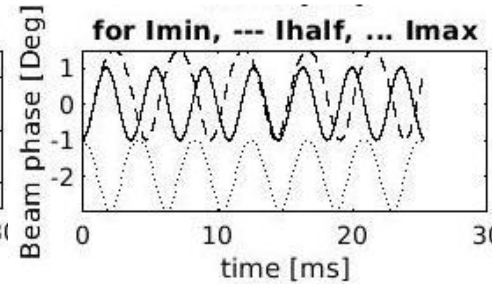
$n\phi_h = n\phi_{h,opt}$ = -12.2 deg

5 cavities, $\beta_h = 2$

$\rightarrow$ Unstable for I_{beam} > 150 mA



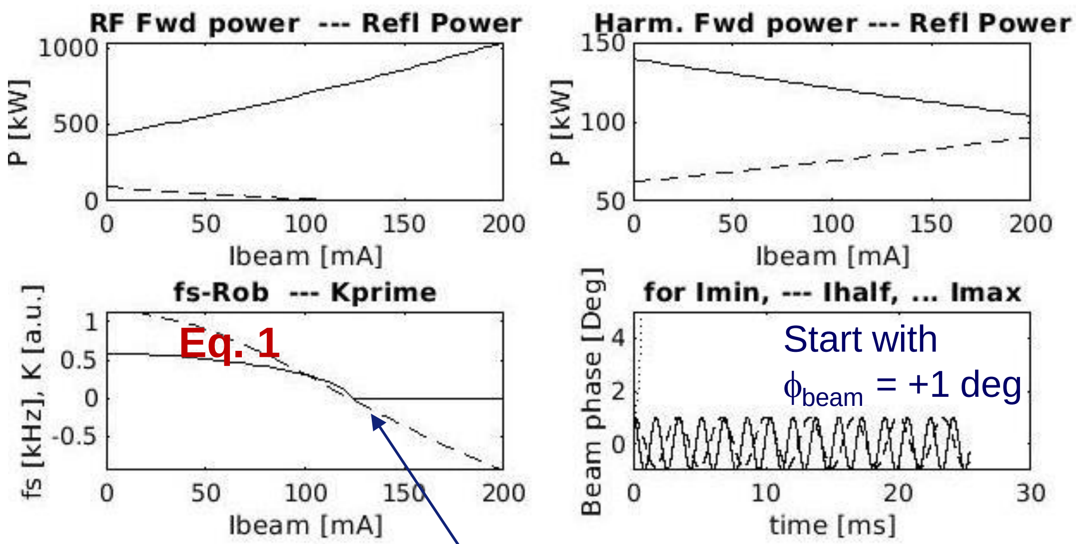




Start with ϕ_{beam} = -1 deg

Threshold at ≈ 150 mA confirmed by numerical integration

ROBINSON DC (2ND TYPE) – INTEGRATION OF SYNCHROTRON EQUATION



V_h = 1.50 MV ($\neq V_{h,opt}$)

$n\phi_h = n\phi_{h,opt}$ = -12.2 deg

5 cavities, β_h = 5

$\rightarrow$ Unstable for I_{beam} > 130 mA

Threshold at ≈ 130 mA confirmed by numerical integration

For active system, Integration of synchrotron equation indicates:

- Robinson stable if Harmonic RF beam loading > Main RF beam loading
  - $\Rightarrow$ Sufficient harmonic cavity impedance,
  - $\Rightarrow$ Sufficient number of harmonic cavities
  - $\Rightarrow$ Upper limit for coupling β_h

DLLRF SYSTEM FOR FAST DIGITAL RF FEEDBACK IS NEEDED

1. To stabilize main RF, one can compute the optimum correction for a fast RF feedback:

$$\begin{pmatrix} \Delta I_{gr} \\ \Delta Q_{gr} \end{pmatrix} = \frac{1}{\cos\psi} \begin{bmatrix} \cos(\psi - \varphi_{tune}) & \sin(\psi - \varphi_{tune}) \\ -\sin(\psi - \varphi_{tune}) & \cos(\psi - \varphi_{tune}) \end{bmatrix} \begin{pmatrix} \Delta I_c \\ \Delta Q_c \end{pmatrix}$$

where $\psi = f(V_c, I_{beam}, N_{cav}, R_s, \beta, \varphi_{tune})$, ($\rightarrow \varphi_{tune}$ = *Load angle, mostly zero)*

2. Alternative: simulate behaviour of a passive cavity

$\rightarrow$ Feedback harmonic voltage phase to lock on beam phase

$\rightarrow$ Results of synchrotron equation integration need to be cross-checked with particle tracking simulations

$\rightarrow$ The two RF feedback approaches need to be included in the simulations and checked