PyAT Developments Towards Harmonic Cavity Simulations

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OVERVIEW

- PyAT overview
- Main cavity beam loading model
- LCBI calculations
- Road map for Harmonic Cavity Simulations

OVERVIEW

- **Accelerator Toolbox (AT) is a 6D tracking code which can perform element by element tracking of particles through a lattice.**
    - Each element has a PassMethod which is written in C.
    - AT is most known as being run with MatLab. But over the past few years, PyAT has become heavily favoured and has received significant development.

- **Single bunch collective effects tracking is possible thanks to the fast_ring function, which reduces the lattice to a few elements.**
    - Reduces to: N_cavities + Linear map + non linear map + diffusion element.

- **Multi bunch collective effects now possible, benchmarking with LCBI and TRW ongoing. All collective effects are parallelized using openmpi.**

- **A beam loading pass method that iteratively computes the beam voltage and updates the cavity voltage and phase has been written and is under test.**
    - The longitudinal resonator wake function is applied over multiple turns, bunch slicing can also be included.

Coordinates

The 6-d phase space coordinates used in AT are as follows $\vec{Z} = \begin{pmatrix} x \\ \frac{p_x}{p_0} = x'(1+\delta) \\ y \\ \frac{p_y}{p_0} = y'(1+\delta) \\ \delta = \frac{(p_z - p_0)}{p_0} \\ c\tau \end{pmatrix}$

p_0 is the reference momentum. τ is the time lag relative to the ideal particle.

RF Cavity definition:

$$\Delta dp = -\frac{V_{rf}}{E}\sin\left(\frac{2\pi f_{rf}}{c}(z - z_{lag}) - \psi\right)$$

Resonator impedance:

$$\mathcal{Z}(f) = \frac{R_s}{1 + jQ\left(\frac{f}{f_r} - \frac{f_r}{f}\right)}$$

Longitudinal resonator wake field definition from Chao:

$$\mathcal{W}(\tau) = \begin{cases} \alpha R_s & \text{for } \tau = 0, \\ 2\alpha R_s e^{-\alpha\tau}\left[\cos(\bar{\omega}\tau) - \frac{\alpha}{\bar{\omega}}\sin(\bar{\omega}\tau)\right] & \text{for } \tau > 0, \end{cases}$$

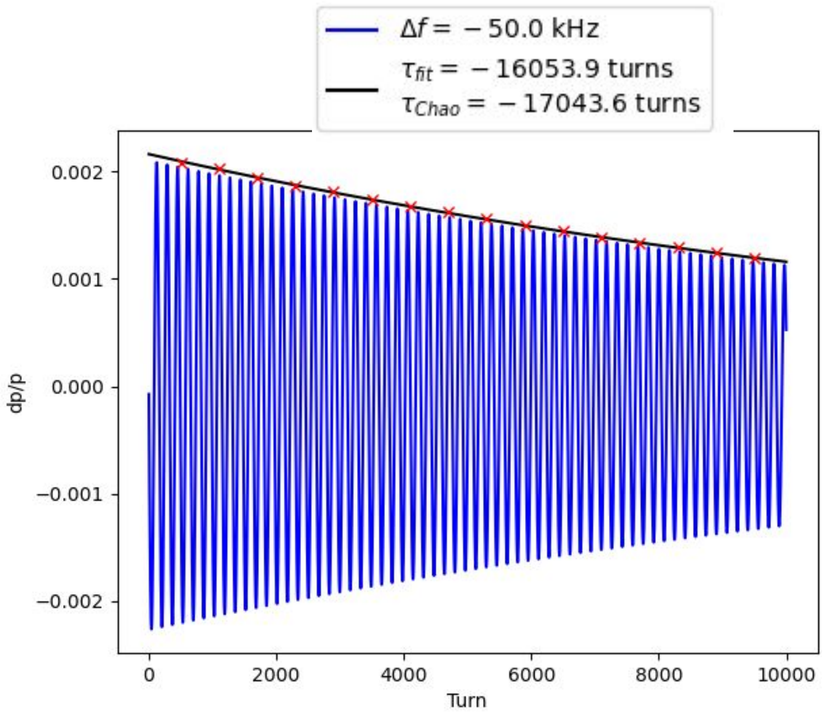
$$\bar{\omega} = \sqrt{\omega_r^2 - \alpha^2} \qquad \alpha = \pi f_r / Q$$

AC ROBINSON INSTABILITY - SINGLE BUNCH RESONATOR BENCHMARKING

- **Slightly modified analytic formula for the growth rate of a particle interacting with a resonator is given below from Chao:**

$$\tau^{-1} \approx \frac{q_e^2 N \,\eta\, h\, \omega_0}{2\, m_e c^2 \gamma T_0^2 \omega_s}\left[\mathrm{Re}\, Z_0^l(h\omega_0 + \omega_s) - \mathrm{Re}\, Z_0^l(h\omega_0 - \omega_s)\right]$$

- **Will use this to benchmark the resonator wake implementation.**
- **For this example, Rs=6 MOhm, Q=4500, Δf=-50kHz, Ib=0.1A, wturns=250, Nslice=1, radiation off.**



BEAM LOADING FORMULA

$$\phi_s = \sin^{-1}\left(\frac{U_0}{V_{rf}}\right)$$

$$\theta = \pi - \phi_s - \phi_L$$

$$V_b = 2I_b R_s$$

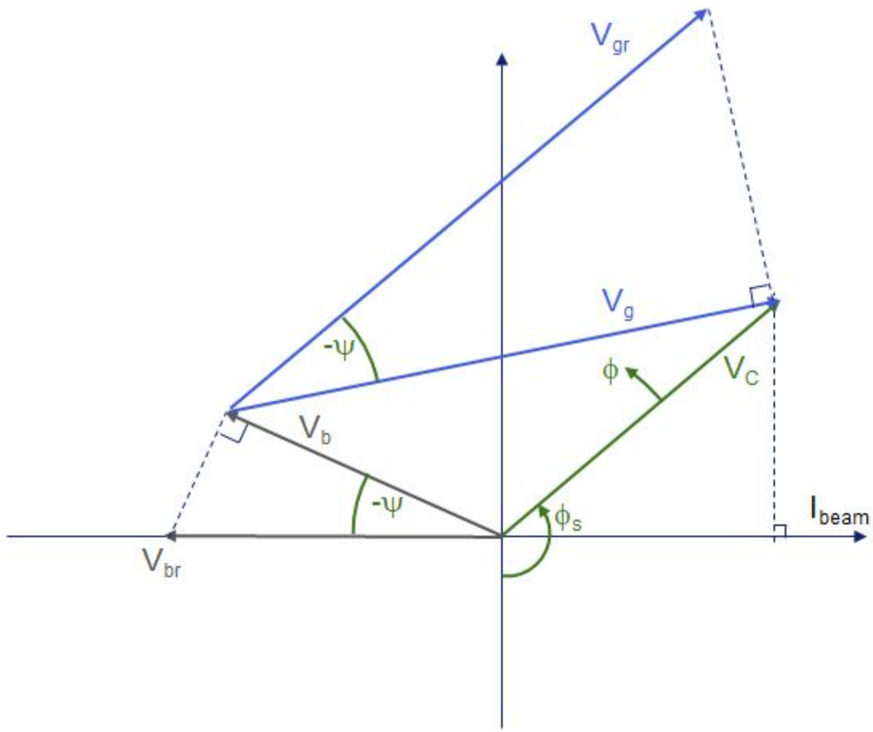
$$x = \frac{b}{\sqrt{a^2 + b^2}}$$

$$a = V_{rf}\cos\phi_s\sin\theta + U_0\cos\theta$$

$$b = V_{rf}\cos\phi_s\cos\theta - (V_b + U_0)\sin\theta$$

$$\psi = \sin^{-1}(x)$$

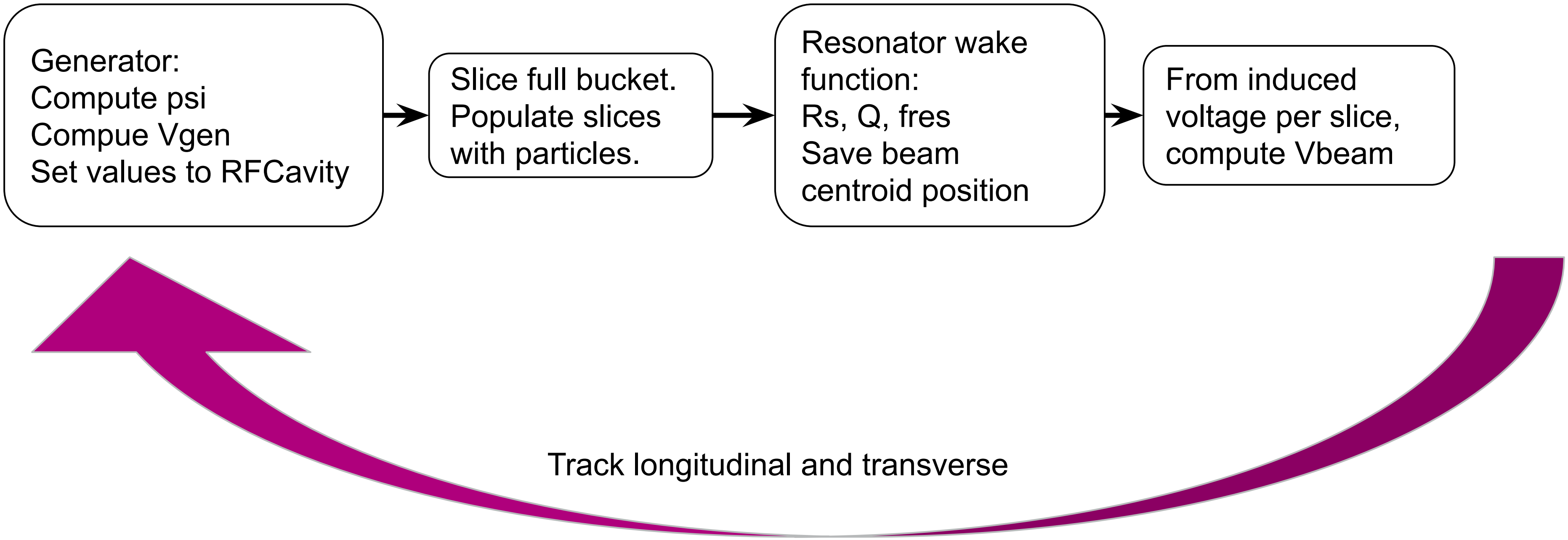
$$V_{gen} = \frac{R_s\cos\psi}{\cos\phi_L}\left(\frac{V_{rf}}{R_s} + 2I_b\sin\phi_s\right)$$



$$Q_s = \frac{Q_{s0}\sqrt{V_{rf}\cos\phi_s + V_b\sin x\cos x}}{\sqrt{V_{rf}\cos\phi_s}}$$

BEAM LOADING FLOW CHART

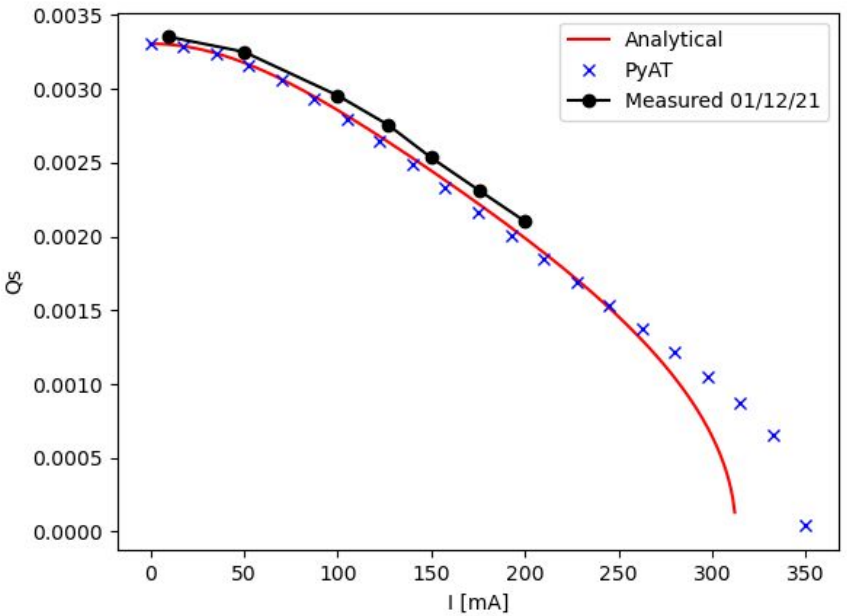
Set_Vrf=6MV,
Vbeam_turn0=2*Ib*Rs



Nslice=1 but the multi-turn wake is applied. wturns is user defined.
The previous wturns centroid positions are stored so Vbeam can be
calculated taking multiple turns into account.

SYNC. FREQUENCY VS CURRENT - SINGLE BUNCH

- Good agreement up to 250 mA.
- Above this the simulation deviates from the analytical case due to the fact we lump all of the current into a single bunch → additional voltage deposition becomes significant at high current.
- Needs multibunch to be fully validated. Full benchmarking ongoing.



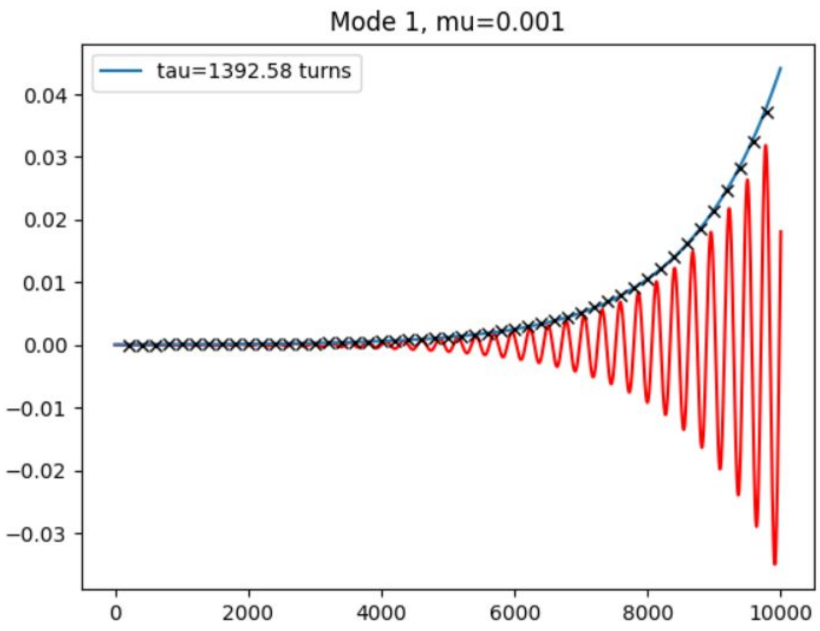
LCBI WITH PYAT - MULTIBUNCH BENCHMARKING

- Current = 200 mA
  Uniform filling, M=992
  Npart_per_bunch = 640 (low)
  Nslice_per_bunch = 1 (ok)

- Longitudinal resonator:
  fres = frf-f0-fs
  Q=1e4
  Rs=2e6

- Analytical growth rate: 272.59s^-1
  Simulated growth rate: 255.08s^-1

- Convergence studies still needed, but implementation is working. Similar discrepancy as single bunch growth rate.



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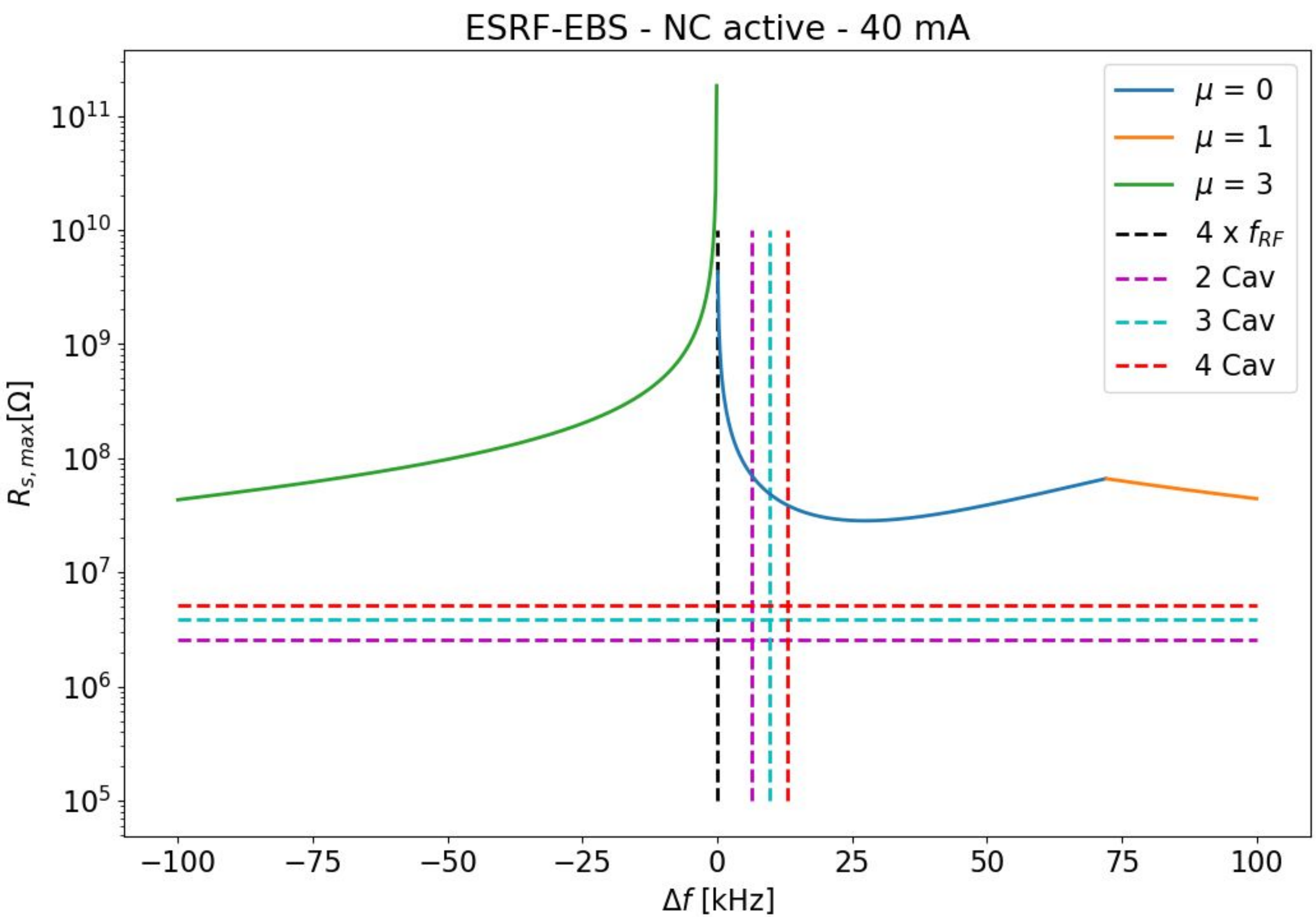
LONGITUDINAL COUPLED BUNCH INSTABILITY

- In previous meeting, Alexis showed some computations of the proximity to the LCBI for different numbers of harmonic cavities.
- He made the script available so we could perform the same analysis for the EBS.

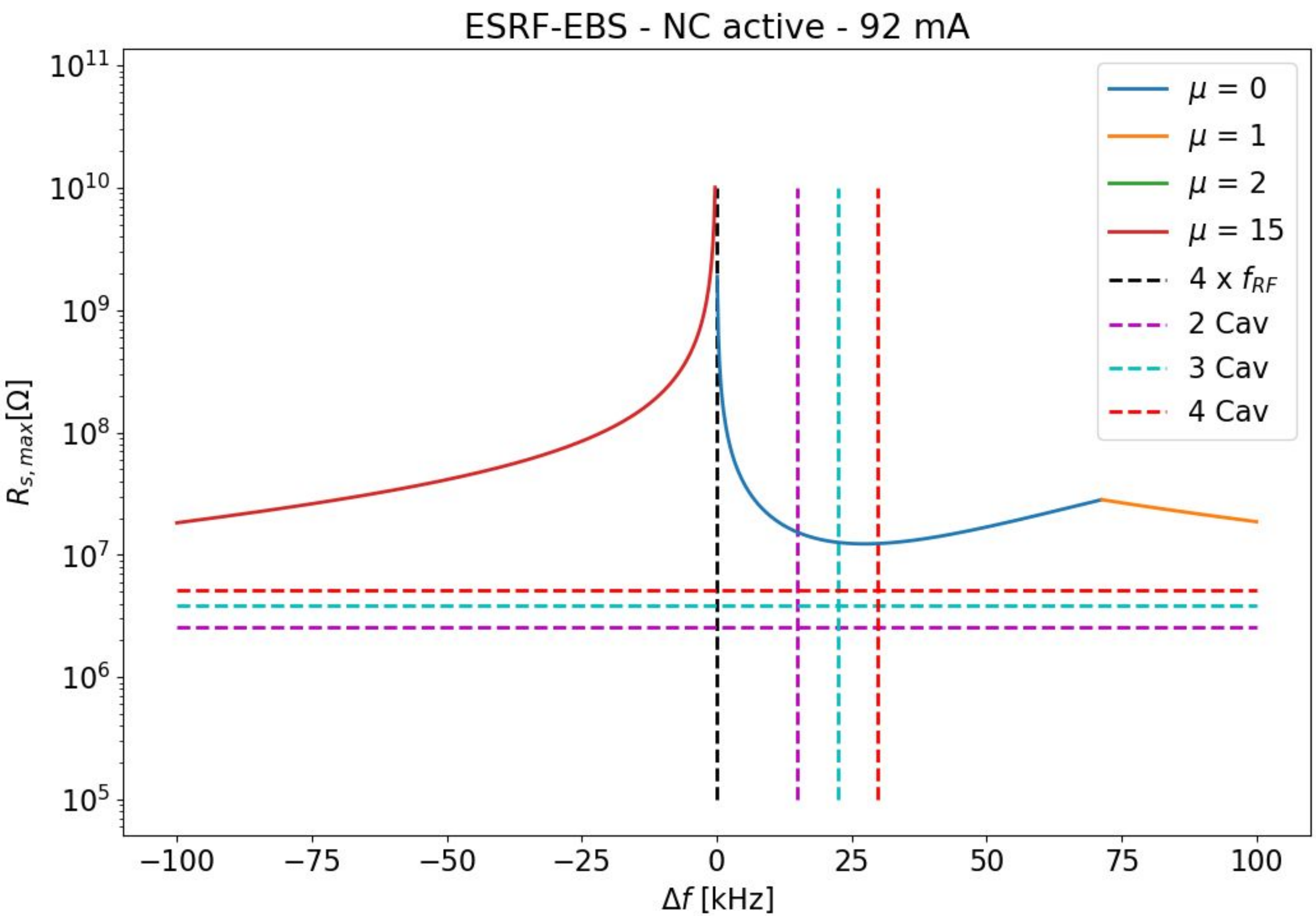
- The EBS parameters can be found in the appendix. For these calculations it assumes $\beta=1$ for the coupling of the harmonic cavity.
- Results shown for the 3 main filling patterns in use at the ESRF.
- Uniform filling is most critical, but in reality we are quite far from all thresholds.

LCBI - 4 x 10 mA

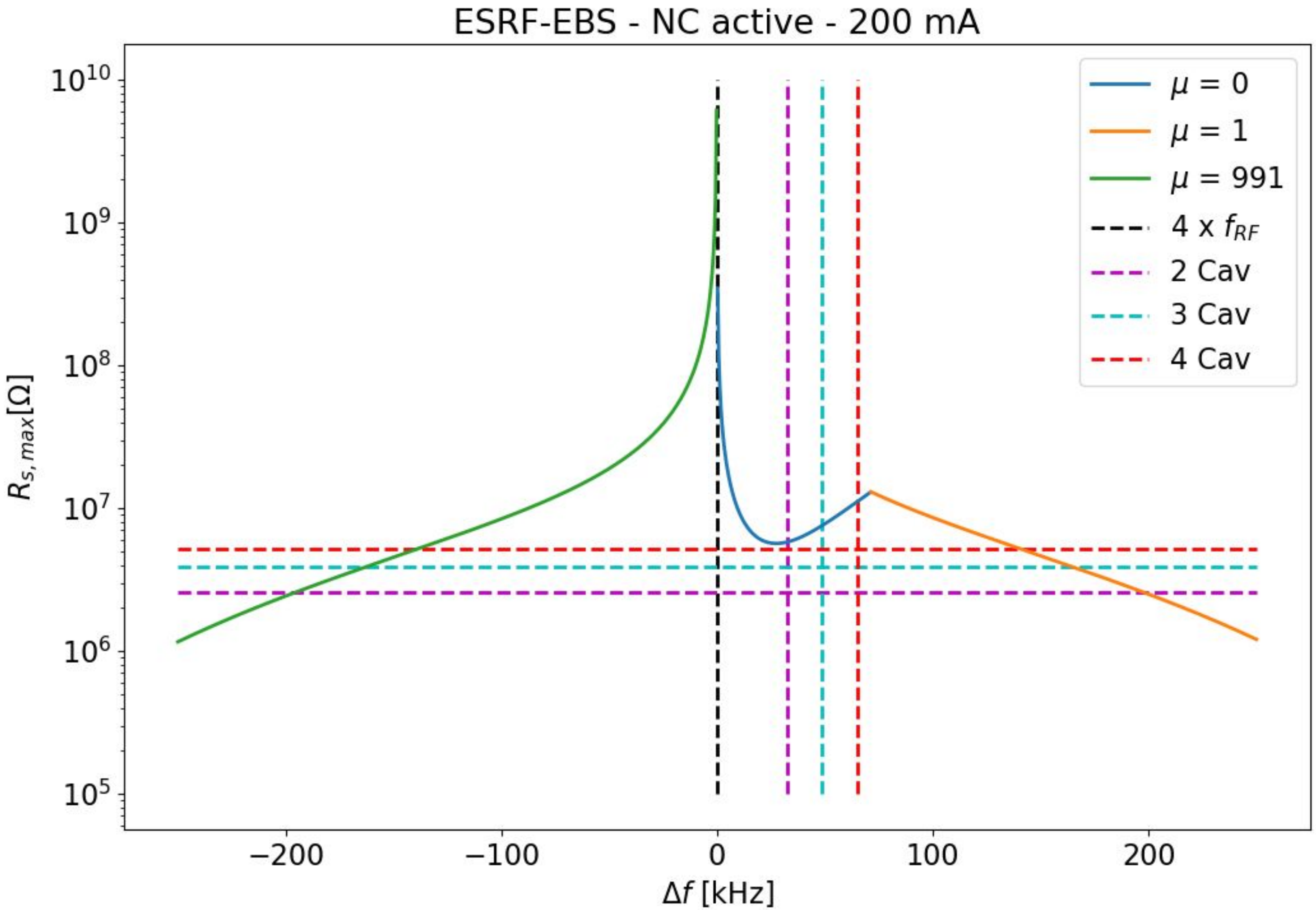


ESRF-EBS - NC active - 40 mA
$R_{s,max}[\Omega]$
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
−100
−75
−50
−25
0
25
50
75
100
Δf [kHz]
μ = 0
μ = 1
μ = 3
4 x f_{RF}
2 Cav
3 Cav
4 Cav

LCBI - 16 BUNCH MODE



LCBI - UNIFORM FILLING



ROAD MAP FOR HARMONIC CAVITY SIMULATIONS

- **We have an almost benchmarked multi-bunch model in PyAT. We are now starting to get to the interesting harmonic cavity simulations.**
  - Still need to recompute synchrotron tune vs current in multi-bunch, requires some modification to the beam loading pass method.

- **We have already some harmonic cavity simulations without beam loading (main or harmonic). Implementation of simple phase loops to recover optimal bunch lengthening.**
  - Nothing interesting to show yet.
  - Has highlighted the question of how this will be implemented in real life i.e. what are our observables / diagnostics? The simulations should reflect reality. Is the flat potential really achievable or do we just go for maximum bunch lengthening?

- **Can anyone share results of other benchmarking problems (now: multibunch without beam loading, later: multibunch with beam loading) so we can benchmark what we have more thoroughly?**

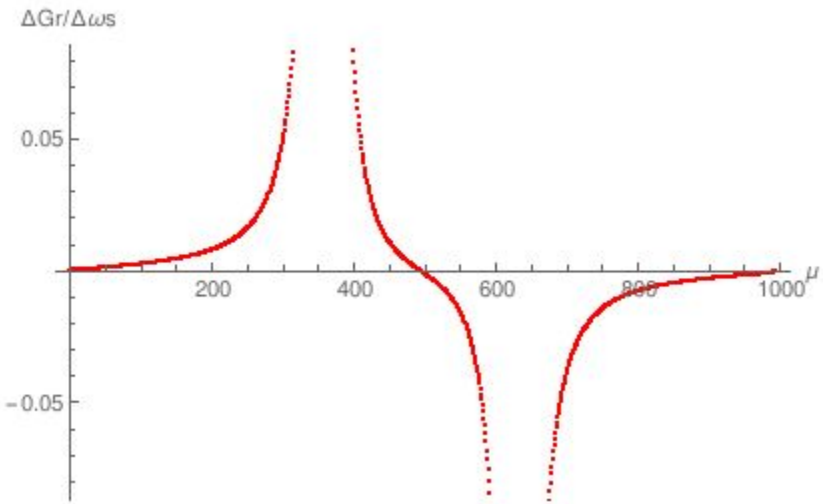
EBS PARAMETERS

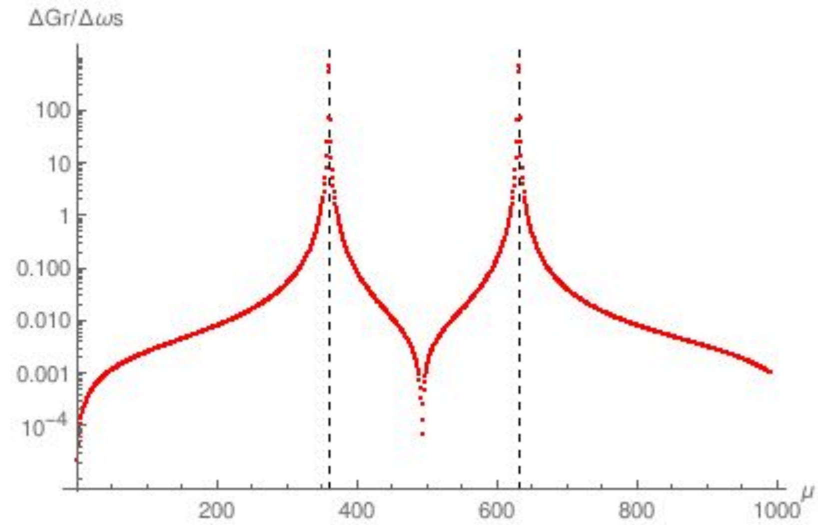
```
m = 4
Q_hc = 29735
Rs_hc = 86.8 * Q_hc # per cavity
beta_hc = 1
QL_hc = Q_hc / (1 + beta_hc)

Vc = 6 MV

h = 992
L = 843.977
E0 = 6e9
particle = Electron()
ac = 8.51173e-05
U0 = 2.53 MeV
taux, tauy, tauz = 8.70405 ms, 13.33987 ms, 9.09085 ms
tunex, tuney = 76.21, 27.34
emitx, emity = 131.84 pm, 5 pm
sigma_0 = 9.744935 ps #13 ps
sigma_delta = 9.3566e-4
```

EXTRA





- **For mode mu=0, the reduced fs should be taken instead of fs0.**
- **However, checking the sensitivity of the growth rate to changes in ws, showed that for mode 0, it is completely insensitive to synchrotron frequency.**
- **Therefore, what we already have is more than accurate.**